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HYSTERESIS CYCLES AND INVARIANCE OF THE *Dst* INDEX FORM DURING GEOMAGNETIC STORM DEVELOPMENT

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Abstract. We have studied the relationship between the *Dst* index and heliospheric parameters during 933 isolated geomagnetic storms observed over the period from 1964 to 2010. The storms were classified by their onset type (sudden or gradual) and intensity (weak, moderate, and strong). We have analyzed the *Dst* index, solar wind, and interplanetary magnetic field (IMF) data accumulated using the epoch superposition method. It is shown that over the time interval of development of varying intensity storms with sudden and gradual onset the trajectory of *Dst* change depending on heliospheric parameters during the main phase of the storms does not coincide with its trajectory during the recovery phase, which is typical of the hysteresis phenomenon. During the storms, *Dst* forms hysteresis cycles with all analyzed solar wind and IMF parameters. The obtained dependences $Dst(B)$, $Dst(B_z)$, $Dst(E_y)$, $Dst(V)$, $Dst(P_{\text{dyn}})$, and $Dst(\beta)$ have the shape of a hysteresis loop during the excitation of weak, moderate, and strong storms. The shape and area of hysteresis loops was found to change depending on heliospheric parameters and storm intensity. It is shown that the shape of the average *Dst* dynamics during the storms does not depend on their intensity, i.e. it is invariant. Invariant behavior is also characteristic of the shape of the average dynamics of heliospheric parameters during the magnetic storms of different intensities. Based on the nonlinear relationship of the *Dst* index with interplanetary parameters and the invariance of the shape of its dynamics, an integral equation of the Volterra type is proposed to describe the *Dst* dependence on solar wind and IMF parameters. The proposed model is suitable for interpreting the results obtained from the experimental study of hysteresis effects associated with phase shifts between changes in *Dst* and heliospheric parameters.

Keywords: geomagnetic storms, solar wind, heliospheric parameters, *Dst* index, hysteresis, invariance.

INTRODUCTION

The hysteresis effect is known be one of the features of solar activity, and not an artifact [Donnelly, 1991; Bachmann and White, 1994]. Currently, there are a large number of publications that show that hysteresis exists between pairs of solar activity indicators. For example, relationships were analyzed between pairs of such solar activity indicators as the sunspot number *W*, the solar radio flux at a wavelength of 10.7 cm *F*10.7, the solar constant *TSI* (Total Solar Irradiation), the index of solar flares, the index of maximum velocity of coronal mass ejections, etc. It has been established that the dynamics of these indicators during the ascending and descending phases of solar activity cycles do not coincide, which is a sign of hysteresis [Bachmann, White, 1994; Özgüç et al., 2012; Bruevich et al., 2018].

In a number of works such as [Dmitriev et al., 2002; Özgüç et al., 2016; Reda et al., 2023], the hysteresis effect was found between solar activity indicators and heliospheric parameters, as well as between solar and geomagnetic activity indices. In particular, during solar cycle 23, hysteresis was observed between geomagnetic indices (A_p, *Dst*) and the maximum velocity of coronal mass ejections [Özgüç et al., 2016]. Hysteresis between the geomagnetic and solar activity indices is typical not only for cycle 23, but also for other solar cycles. For inctance, Kurazhkovskaya, Kurazhkovskii [2023] have found the hysteresis effect between the geomagnetic activity indices (A_p, *Dst*) and heliospheric parameters in solar cycles 21–24. Moreover, the A_p and *Dst* indices form hysteresis loops not only with all key solar wind (SW) and interplanetary magnetic field (IMF) parameters, but also with their combinations, for example, with the SW dynamic pressure P_{dyn}, the plasma parameter β equal to the ratio of thermal pressure to magnetic pressure [Kurazhkovskaya, Kurazhkovskii, 2023]. Thus, at least for the past four completed solar cycles, hysteresis is clearly observed in the solar, interplanetary, and geomagnetic activity dependences at time intervals equal to the solar cycle.

Nonetheless, hysteresis is not only a typical feature of ~11-year solar cycles, but is also recorded at shorter time intervals comparable to the duration of geomagnetic storms (3 days on average) [Ptitsyna et al., 2021; Kurazhkovskaya, Kurazhkovskii, 2023; Danilova et al., 2024]. Kurazhkovskaya, Kurazhkovskii [2023] have shown that during geomagnetic storms the average dependence of *Dst* on β has the form of a hysteresis loop. Preliminary studies on the relationship between the *Dst* index and other heliospheric parameters during geomagnetic storms have found that *Dst* forms hysteresis loops with the SW dynamic pressure P_{dyn} and the IMF B_z component [Zotov et al., 2024]. When examining the September 7–8, 2017 strong magnetic storm, signs of hysteresis were detected in the *Dst* dependence on *B* and B_z, and the SW electric field component E_y [Danilova et al., 2024]. The question as to how the shape and size of hysteresis loops between *Dst* and the SW and IMF parameters change depending on the intensity of geomagnetic storms is still open.

In this paper, we analyze hysteresis effects between the *Dst* index and heliospheric parameters during isolated geomagnetic storms of varying intensity with sudden and gradual onset, examine features of the obtained hysteresis loops, and propose a mathematical model of the effect.

1. DATA AND ANALYSIS METHOD

To study the relationship between the *Dst* index and IMF parameters during geomagnetic storms, we have used the catalog of geomagnetic storms presented on the website of the World Data Center for Solar-Terrestrial Physics (Moscow) [http://www.wdcb.ru/stp/geomag/geomagnetic_storms.ru.html]. In addition, we have employed the hourly averages of the SW and IMF parameters and the *Dst* index presented in the OMNI database [https://spd f.gsfc.nasa.gov/pub/data/omni/low_res_omni/].

The catalog contains data for the period 1950–2010 for geomagnetic storms of two categories: with sudden and gradual onset. We took information about geomagnetic storms that occurred in 1964–2010 from the catalog since there was the most complete satellite data in the OMNI database for this period. The sample we obtained includes 933 isolated geomagnetic storms: 288 with sudden onset and 645 with gradual onset.

As inferred from [Taylor et al., 1994; Loewe, Prölss, 1997], minimum *Dst* can be utilized as an indicator of storm intensity. According to minimum *Dst*, storms are divided into at least three classes: weak (–30 nT≥Dst_{min}>–50 nT), moderate (–50 nT≥Dst_{min}>–100 nT), and strong (–100 nT≥Dst_{min}). We have classified storms with sudden and gradual onset by intensity.

To construct average hysteresis loops, we have used hourly values of *Dst*, SW and IMF parameters obtained by the epoch superposition method. We have examined the SW and IMF parameters that in any case play an important role in developing geomagnetic storms, namely the IMF modulus *B* and northward-southward component B_z, the azimuthal electric field component E_y, the velocity *V*, the dynamic pressure P_{dyn}, and the SW parameter β (β is equal to the ratio of thermal pressure to magnetic pressure $\beta = NkT/(B^2/8\pi)$). The OMNI database uses the formula

$$\beta = \left(\frac{4.16T}{10^5} + 5.34\right)\frac{N_p}{B^2},$$

where *T* is the temperature (K); N_p is the proton density [cm^{-3}]; *B* is the magnetic field strength [nT].

The dynamics of the *Dst* index and heliospheric parameters was studied for 48 hrs before and 168 hrs after the storm onset, indicated in the catalog. Information on the number of isolated storms with sudden and gradual onset, classified as weak, moderate and, strong, as well as their minimum t_{min}, maximum t_{max}, and mean duration t_{mean} (obtained from the catalog) is presented in Table.

Table indicates that the duration of the storms varies widely: t_{min} varies from 6 to 13 hrs; and t_{max}, from 149 to 182 hrs. The average dependence of the *Dst* index on the SW and IMF parameters was studied in a time interval corresponding to the average duration of storms of each class.

2. RESULTS AND DISCUSSION

2.1. Dynamics of geomagnetic activity and heliospheric parameters during storms with sudden and gradual onset

Before proceeding to the study of the hysteresis effect between *Dst* and heliospheric parameters, we analyze SW and IMF conditions during varying intensity storms. The epoch superposition method is widely used to study heliospheric conditions during geomagnetic storms [Taylor et al., 1994; Loewe, Prölss, 1997; Lyatsky, Tan, 2003; Zhang et al., 2006; Yermolaev et al., 2007; Yermolaev et al., 2010a; Hutchinson et al., 2011]. Note that most studies deal with conditions in the interplanetary medium during moderate and strong storms (with *Dst*≤–50 nT). In [Kurazhkovskaya et al., 2021], we have examined the dynamics of the *Dst* index and the β parameter not only for moderate and strong storms, but also for weak storms with sudden and gradual onset, but the comparison of the behavior of the *Dst* index with the behavior of other parameters was made without classifying storms by intensity. In this paper, we examine the features of changes in interplanetary medium parameters for storms with sudden and gradual onset depending on their intensity.

Figure 1 shows the time variation in hourly averages of *Dst*, *B*, B_z, E_y, *V*, P_{dyn}, and β during isolated storms

Duration, hr	Storms with sudden onset			Storms with gradual onset		
	weak N=50	moderate N=133	strong N=105	weak N=278	moderate N=308	strong N=59
t_{min}	10	6	9	8	11	13
t_{max}	150	149	166	181	182	157
t_{mean}	39	38	45	40	46	49

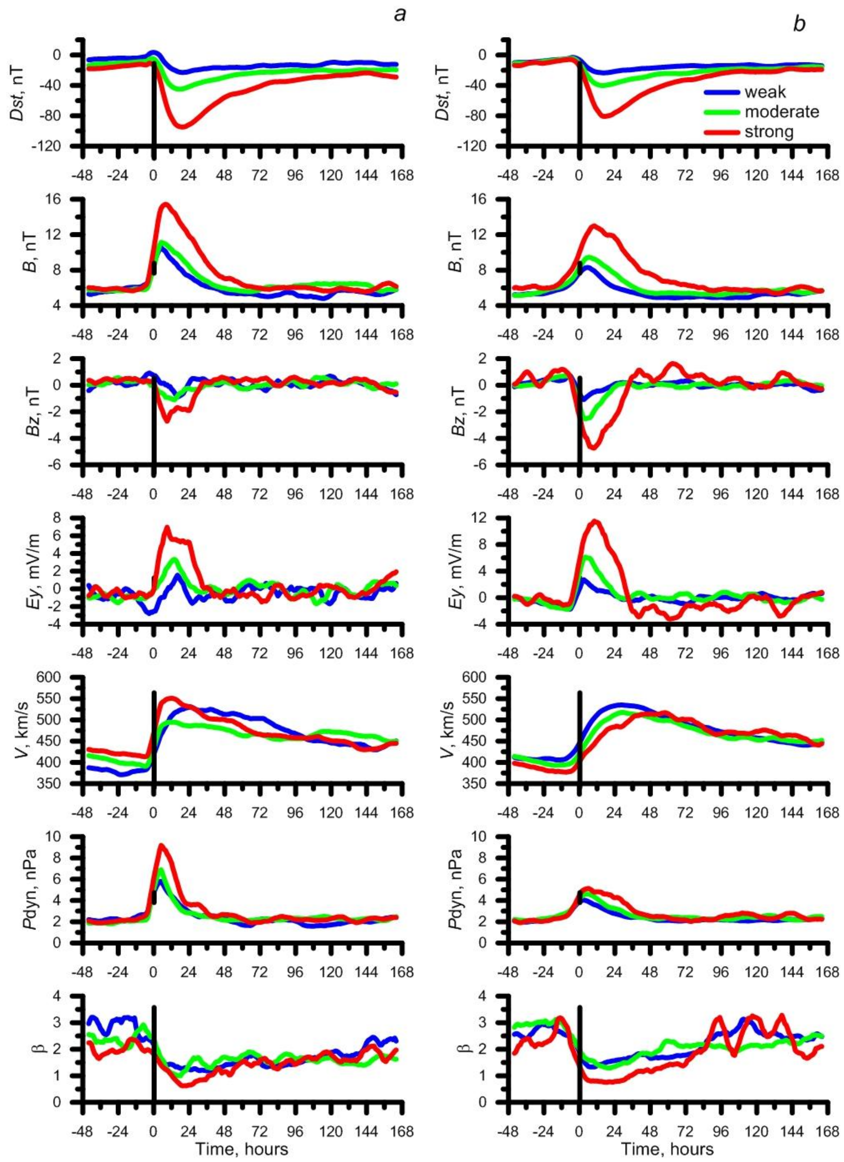

Figure 1. Dynamics of hourly averages of the *Dst* index, the IMF modulus *B* and B_z component, the electric field component E_y, the velocity *V*, the dynamic pressure P_{dyn}, and the SW parameter β during isolated storms of varying intensity with sudden (*a*) and gradual (*b*) onset

storms with sudden (*a*) and gradual (*b*) onset. Here and in the following figures, weak storms are indicated in blue; moderate storms, in green; and strong storms, in red. In fact, Figure 1 illustrates the typical behavior of the *Dst* index, IMF and SW parameters during storms with sudden and gradual onset. The dynamics of *Dst* during storms with sudden and gradual onset is qualitatively similar regardless of their intensity. During storms with sudden onset, peak absolute *Dst* during the main phase is much larger than that during storms with gradual onset. This pattern is observed both for weak and for moderate and strong storms, but strong storms exhibit large peak absolute *Dst* as compared to weak and moderate storms.

Plots of the IMF B_z component show that during storms of all classes, ~1–2 hrs before the start of the storm, B_z deviates to the south (changes direction from positive to negative), and then, after reaching a minimum, gradually returns to its original level. In contrast to peak absolute *Dst*, peak absolute B_z during the main phase in the case of storms with sudden onset is smaller than in the case of storms with gradual onset. The average peak absolute B_z component depends on storm intensity. The highest absolute B_z is observed during strong storms. Since E_y is closely related to B_z, its dynamics mirrors the behavior of B_z. It is characteristic that peak values of both E_y and B_z are larger during storms with gradual onset.

Near the storm onset, averages of the IMF modulus, the SW velocity and dynamic pressure begin to increase, reaching a maximum during the main phase, and then gradually decrease during the recovery phase. Dynamics of these parameters is identical when varying intensity storms with sudden and gradual onset are excited. Unlike moderate and strong storms, the occurrence of weak storms is associated with small increases in peak values of |*B*|, *V*, and P_{dyn}.

One of the features of the behavior of the parameter β is a qualitative coincidence of its dynamics with the dynamics of the *Dst* index, which has been pointed out in [Kurazhkovskaya et al., 2021]. The values of β begin to decrease a few hours before the start of the storm, approaching 1 during the main phases of weak and moderate storms. During strong storms, β becomes less than 1 in the main phase. During the recovery phase, β returns to the background level.

In general, the average dynamics of *Dst* and heliospheric parameters from –48 to +168 hrs relative to the moment of the storm onset is similar to the development of isolated storms with sudden and gradual onset. A similar time variation in heliospheric parameters was observed in works in which the moment of the storm onset was taken as a zero point, for example [Taylor et al., 1994; Yermolaev et al., 2007; Yermolaev et al., 2010a; Hutchinson et al., 2011; Ahmed et al., 2024]. At the same time, the peak values of the heliospheric parameters (see Figure 1) differ significantly when storms of varying intensity are excited. Near the storm onset, regardless of the type of onset, moderate and strong storms exhibit larger increases in SW and IMF parameters than weak storms.

A comparison between conditions of storms with sudden and gradual onset has confirmed the previously discovered pattern [Taylor et al., 1994; Loewe, Prölss, 1997], namely, the dependence of peak B_z and P_{dyn} on the type of storm onset. Referring to Figure 1, during storms with sudden onset the peak values of P_{dyn} are larger, and the peak absolute values of B_z are smaller than during storms with gradual onset, regardless of their intensity.

2.2. Hysteresis cycles between *Dst* and SW and IMF parameters

Examine *Dst* dependences on SW and IMF parameters whose dynamics during storms is illustrated in Figure 1. The hourly averages of heliospheric parameters, obtained by the epoch superposition method, have been previously smoothed by a moving average of 15 points. The dependences *Dst*(B), *Dst*(B_z), *Dst*(E_y), *Dst*(V), *Dst*(P_{dyn}), and *Dst*(β) were plotted for the time intervals of the average duration of weak, moderate, and strong storms (see Table) from the storm onset (zero point). Figures 2 and 3 show the obtained dependences for varying intensity storms with sudden (*a*) and gradual (*b*) onset. The storm onset is marked with an asterisk. The average *Dst* index is seen to form a loop-like relationship with all analyzed parameters. This means that the curve of *Dst* as function of heliospheric parameters for the storm main phase does not coincide with that for the recovery phase, which is typical of hysteresis.

In other words, during geomagnetic storms *Dst* and SW and IMF parameters vary cyclically, or form hysteresis cycles. One of the conditions for the occurrence of hysteresis loops is a phase shift between the analyzed parameters, and the width of the hysteresis loops reflects these phase shifts (or time delays) between *Dst* and interplanetary medium parameters in this case. The dependences *Dst*($|B|$), *Dst*(B_z), *Dst*(E_y), *Dst*(V), *Dst*(P_{dyn}), and *Dst*(β) illustrated in Figures 2, 3 reflect the response of *Dst* to variations in the SW and IMF parameters during storms. In addition, these dependences indicate time shifts between the disturbance of the magnetosphere and the heliospheric parameters during the storms, as well as the nonlinear relationship of *Dst* with the SW and IMF parameters.

It is characteristic that hysteresis between *Dst* and interplanetary medium parameters exists regardless of the storm intensity. However, areas of hysteresis loops depend on the storm intensity: the largest loop areas are observed during strong storms. The size of the *Dst* hysteresis loops also depends on the SW and IMF parameters considered. By this means *Dst* forms relatively wide loops with almost all parameters, except β, for storms of all classes, and the SW velocity only for weak storms. Note that the width of the hysteresis loops between *Dst* and B_z is narrower, and between *Dst* and P_{dyn} is wider during storms with sudden onset than during storms with gradual onset. This fact confirms the assumption of Taylor et al. [1994] that B_z is a parameter controlling the excitation of storms with gradual onset, and the SW dynamic pressure plays a dominant role in storms with sudden onset.

Hysteresis loops (see Figures 2, 3) differ in the direction of rotation. For example, during a storm of any type in the hysteresis loops formed by *Dst* with the IMF modulus B and the electric field component E_y, the changes occur clockwise; whereas in the hysteresis loops of *Dst*(B_z) and *Dst*(β), counterclockwise. Note that Danilova et al. [2024], when studying the hysteresis effect between *Dst* and interplanetary parameters during the September 7–8, 2017 storm, obtained similar rotation directions: clockwise in *Dst*(B) and *Dst*(E_y), and counterclockwise in *Dst*(B_z) and *Dst*(β). Apparently, this similarity is not accidental, and the dependences of interest reflect the magnetospheric response to changes in SW and IMF conditions when storms are excited.

Thus, hysteresis between *Dst* and heliospheric parameters during geomagnetic storms was found both in the average statistical analysis of geomagnetic storms in this work and in the study of individual events [Danilova et al., 2024].

2.3. Invariance of the dynamics of the *Dst* index and heliospheric parameters

As noted in Subsection 2.1, the dynamics of the *Dst* index is qualitatively similar when weak, moderate, and strong storms with sudden and gradual onset are excited.

The shape of the averaged curve of *Dst* values during storms with sudden and gradual onset does not depend on their intensity, hence the average dynamics of the *Dst* index is invariant. The invariant behavior is also typical of the heliospheric parameters during magnetic storms.

Figure 4 displays averaged curves of *Dst* values, IMF modulus, and SW dynamic pressure for the interval from 24 hrs before and to 72 hrs after the start of the storm. For ease of comparison, the *Dst*, B, and P_{dyn} curves were normalized to their maximum values. We can see (Figure 4) that the behavior of *Dst* and heliospheric

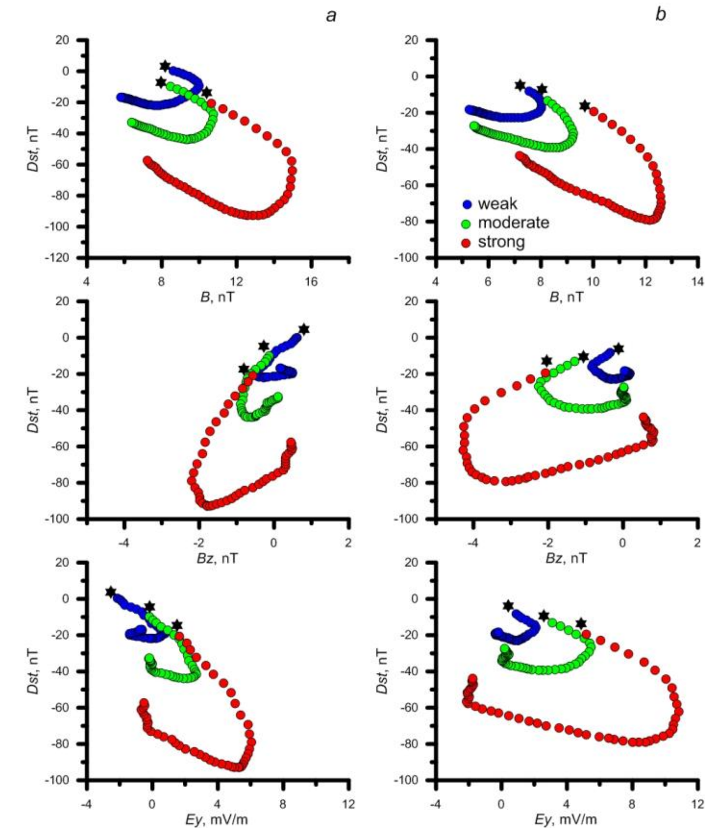

Figure 2. Hysteresis loops formed by geomagnetic activity (*Dst* index) with IMF parameters *B* and B_z, and the SW electric field component E_y during weak, moderate, and strong geomagnetic storms with sudden (*a*) and gradual (*b*) onset

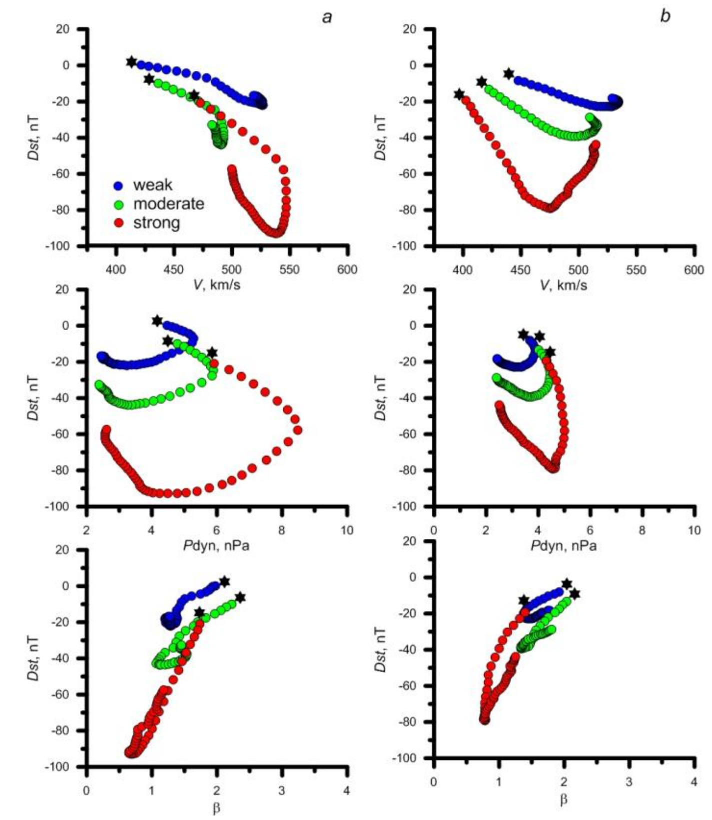

Figure 3. Hysteresis effect for *Dst* as a function of SW velocity *V*, dynamic pressure P_{dyn}, and plasma parameter β during weak, moderate, and strong geomagnetic storms with sudden (*a*) and gradual (*b*) onset

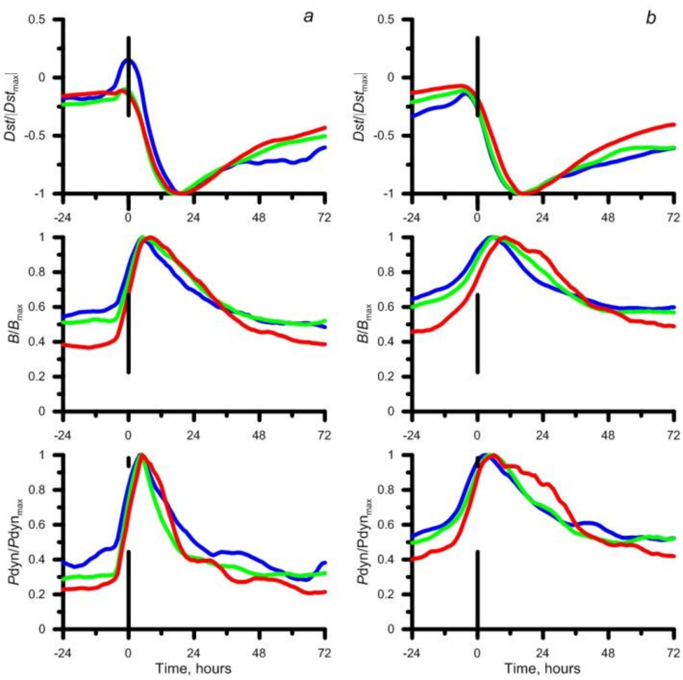

Figure 4. Invariance of average dynamics of *Dst* and heliospheric parameters during weak, moderate, and strong isolated geomagnetic storms with sudden (*a*) and gradual (*b*) onset. The *Dst*, *B*, and P_{dyn} curves are normalized to their maximum values

parameters during the main and recovery phases is similar and independent of the storm intensity, i.e. it is invariant. From Figure 4 it follows that during storms of all types the average dynamics of the *Dst* index is almost identical.

Hutchinson et al. [2011] have analyzed the average dynamics of the geomagnetic index *SYM-H* (*SYM-H* is an analog of *Dst*, but differs from it in time resolution) during weak, moderate, and strong storms. Note that the behavior of *SYM-H* for storms of different types was also similar regardless of their intensity. Moreover, as follows from the results obtained in [Dremukhina et al., 2019], the shape of time variation in *Dst* during storms triggered by different interplanetary drivers is also similar, regardless of the driver type. Thus, a permanent feature of the *Dst* index is the invariance of dynamics regardless of the intensity and type of storm onset, as well as the disturbance source. A similar feature is characteristic of the dynamics of heliospheric parameters during geomagnetic storms.

2.4. Mathematical model of hysteresis dependence of *Dst* on heliospheric parameters during storms

In solar-terrestrial physics, loop-like dependences in solar, interplanetary, and geomagnetic activity are commonly denoted by the term "hysteresis" [Dmitriev et al., 2002; Özgüç et al., 2016; Kurazhkovskaya, Kurazhkovskii, 2023], borrowed from magnetism. In particular, we have shown that the geomagnetic activity index *Dst* forms hysteresis loops with heliospheric parameters during geomagnetic storms. Two features of the development of geomagnetic storms have been found: 1) the hysteresis effect between *Dst* and heliospheric parameters; 2) invariance of the average dynamics of *Dst* and interplanetary medium parameters, which are typical of weak, moderate, and strong storms. Moreover, the hysteresis effect between *Dst* and heliospheric parameters is observed for storms with both sudden onset caused by coronal mass ejections and gradual onset caused by high-speed SW streams from coronal holes [Taylor et al., 1994; Borovsky, Denton, 2006; Obridko et al., 2013]. Since the hysteresis effect between *Dst* and heliospheric parameters during geomagnetic storms exists regardless of their intensity, it can also be considered as an invariant characteristic of storms.

The *Dst* index describing the intensity of the magnetospheric ring current and the strength of storms is known to be one of the main morphological criteria for geomagnetic storms; therefore, numerous studies put emphasis not only on *Dst* features, but also on its modeling. Attempts to develop mathematical models to describe *Dst* dependence on interplanetary medium parameters have repeatedly been made starting with the pioneer work [Burton et al., 1975]. For example, Ji et al. [2012] have overviewed six models. Mathematical modeling of the relationship of the *Dst* index with the SW and IMF parameters is generally based on the analysis of the correlation between them. In our view, taking into account two *Dst* features, namely the hysteresis dependence of *Dst* on heliospheric parameters and the invariance of the average dynamics of *Dst*, we can make use of an alternative approach to describing the *Dst* dependence on the SW and IMF parameters.

A mathematical theory of hysteresis in ferromagnetic materials was first proposed in [Kostitsyn, 1924]. By drawing an analogy between the processes in ferromagnetic materials driven by a magnetic field and the SW- and IMF-induced processes in the magnetosphere during storms, we can adapt the integral hysteresis equation of the Volterra type [Kostitsyn, 1924] for mathematical description of the hysteresis relationship between *Dst* and heliospheric parameters. Before proceeding to the integral hysteresis equation, we apply the scale transformation to the averaged *Dst* index [Donner, Balasis, 2013]:

$$\bar{D}st(t) = \left(Dst(t) - Dst_{\min}\right) / \left(Dst_{\max} - Dst_{\min}\right). \quad (1)$$

We obtain the invariant behavior of *Dst* for all geomagnetic storm phases. Note that Expression (1) allows us to perform scale transformation of any heliospheric parameter. In Figure 5 are curves of *Dst* and $P_{\rm dyn}$ obtained after scale transformation (1) for weak, moderate, and strong isolated geomagnetic storms with sudden and gradual onset. Figure 5 demonstrates the invariance of the average dynamics of *Dst* and $P_{\rm dyn}$ regardless of storm intensity.

With scale transformation of *Dst*, the integral equation can be represented as follows:

$$\bar{D}st(t) = \mu F\left[SW(t)\right] + \\ +\lambda \int_{t_0}^{t} K(t-\tau)\bar{D}st(\tau)\,d\tau, \quad (2)$$

where $\bar{D}st(t)$ is the averaged geomagnetic activity index; $SW(t)$ is an averaged SW and IMF parameter (see, e.g., Figure 5); $F[SW(t)]$ is the rate of SW energy dissipation into the magnetosphere; μ and λ are coefficients; $K(t-\tau)$ is the kernel of integral equation. Using the Laplace transform, solve the integral equation [Krasnov, 1975]:

$$\bar{D}st(t) = (1/2\pi i) \int_{s-i\infty}^{s+i\infty} \frac{F(p)}{1-\lambda K(p)} \exp(pt)\,dp, \quad (3)$$

where $F(p)$ and $K(p)$ are respectively Laplace images of original $F(t)$ and $K(t-\tau)$, $p=s+i\,\sigma$.

The function *F* describes SW energy dissipation in the magnetosphere; and *K*, the internal dynamics of the magnetosphere per se. Since scale transformation (1) leads to invariance of the dynamics of $\bar{D}st(t)$ and heliospheric parameters during geomagnetic storms, in view of relation (3) the SW geoeffectiveness is actually determined by the dynamics of *F* and *K*. Proposed model (2) provides a universal approach to studying geomagnetic storms. However, in our opinion, there are no generally accepted physical models that would allow us to separate the processes caused by the solar wind energy dissipation in the magnetosphere and the internal energy of the magnetosphere per se.

A similar conclusion follows from the Burton equation [Burton et al., 1975]. If we take into account the scale invariance of the *Dst* index and heliospheric parameters, the solution of the equation

$$dDst/dt = Q(t) - Dst/\tau \quad (4)$$

can be represented as

$$Dst = \exp(-t/\tau)\int_0^t \exp(p/\tau) Q(p)\,dp. \quad (5)$$

Here, $Q(t)$ is the rate of energy injection into the ring current; τ is the time of ring current decay. The injection rate $Q(t)$ is related not only to the energy input into the ring current, but also to other current systems in the magnetosphere. Researchers have proposed various processes responsible for the SW energy input into the magnetosphere and the internal dynamics of the magnetosphere depending, for example, on magnetopause currents, generated by the SW pressure, on SW velocity, and on IMF parameters [O'Brien, McPherron, 2000; Vasyliūnas, 2006; Asikainen et al., 2010; Guglielmi, 2016]. Additional magnetospheric and ionospheric current systems can contribute to $Dst(t)$ and should be taken into account in Equation (4), but the relationship of these additional currents with SW parameters is rather difficult to estimate [Love, Mursula, 2024].

It follows from the above that with the existing approaches the Barton and Volterra models for describing a geomagnetic storm do not allow us to unambiguously separate the dynamics of SW energy input and the internal

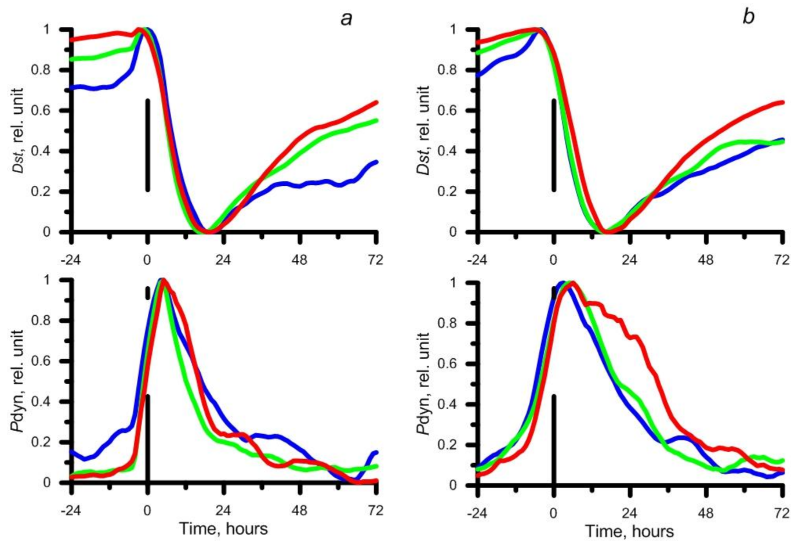

Figure 5. Invariance of the average dynamics of *Dst* and $P_{\rm dyn}$ after scale transformation (1) during weak, moderate, and strong isolated geomagnetic storms with sudden (*a*) and gradual (*b*) onset

dynamics of Earth's magnetosphere. The dynamics of SW energy input into the magnetosphere usually refers to the appearance of geoeffective conditions in SW and IMF, which initiate disturbances in the magnetosphere and in particular the generation of magnetic storms.

Note that the term "geoeffectiveness" is understood in two ways in the literature. For example, Yermolaev et al. [2010b] provide complete statistics of large-scale SW streams and estimate their geoeffectiveness as the probability of occurrence of a magnetic storm, i.e. the ratio of the number of events that caused the magnetic storm to the total number of events of this type. The effectiveness of various interplanetary drivers varies depending on SW stream type and can be estimated as the ratio of measured energy output to estimated energy input [Yermolaev et al., 2012]. In another case, geoeffectiveness is understood as the effectiveness of storm generation by an interplanetary parameter whose correlation with the behavior of the *Dst* index is quite high. The amount of energy entering the magnetosphere and triggering magnetospheric disturbances, as follows from [Yermolaev et al., 2012], is largely determined by the geoeffectiveness of magnetic storm drivers, as well as the internal dynamics of the magnetosphere. To date, there are no methods for assessing the contribution of the internal dynamics of the magnetosphere to geoeffectiveness. Thus, since the dependence of *Dst* on SW parameters is hysteresis, it is impossible to estimate the behavior of the kernel *K* of Equation (2) without hypotheses describing the internal dynamics of the magnetosphere. Without going into detail on possible approaches to solving this problem, we mention a number of works, for example [Zotov et al., 2008; Zotov, Klain, 2017; Balasis et al., 2011, 2018; Johnson et al., 2018], which analyze various models of the internal dynamics of the magnetosphere.

In our view, the proposed model of geomagnetic storm generation, which takes into account hysteresis effects and *Dst* invariance, can adequately describe the dynamics of the magnetosphere and hence can be used for a more accurate theoretical description of the dynamics of Earth's magnetosphere, which will help improve the reliability of space weather forecasting and modeling methods.

CONCLUSION

By studying the average statistical relationship of the *Dst* index with heliospheric parameters, we have identified two effects inherent in geomagnetic storms regardless of the type of their onset and intensity: 1) the hysteresis effect between *Dst* and heliospheric parameters; 2) the invariance of the average dynamics of *Dst* and interplanetary medium parameters during storms. Based on the nonlinear relationship of the *Dst* index with interplanetary parameters and the *Dst* invariance, we proposed an integral equation of the Volterra type to describe the *Dst* dependence on solar wind parameters. The proposed model is suitable for interpreting the results of experimental study of hysteresis effects associated with phase shifts between variations in *Dst* and SW parameters.

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